ICTP Diploma Program in Basic Physics

Advanced electromagnetism

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F IRST answer the multiple-choice questions marking your answers. Then solve the three problems. You are expected to solve all of them. Use extra sheets for your calculations and write below the questions only the crucial steps and the answers. Please, write clearly and in block letters. You have one hours.

MULTIPLE-CHOICE QUESTIONS

- Electromagnetic radiation is generated by:
 - (a) any electric current in a wire
 - (b) any electric charge at rest
 - (c) any accelerated electric charge
 - (d) any magnetic field non-uniform in space
- The magnetic field inside a long solenoid is:
 - (a) inversely proportional to the radius of the solenoid
 - (b) uniform and proportional to the current
 - (c) inversely proportional to the radius of the solenoid and directly to the current
 - (d) uniform and independent of the current
- In crossing a boundary surface, the normal component of the electric \vec{D} field:
 - (a) remain unchanged
 - (b) changes by an amount equal to the surface charge density
 - (c) changes by an amount equal to the tangential component of the electric current density
 - (d) changes by an amount equal to the normal component of the electric current density
- The continuity equation states that the change in charge density $\partial \rho / \partial t$ is equal to:
 - (a) $\vec{\nabla} \cdot \vec{j}$ (\vec{j} being the current density)
 - (b) $-\vec{\nabla}\cdot\vec{j}$
 - (c) $\vec{n} \cdot \vec{j}$ (*n* being normal to the surface of the volume)
 - (d) $\vec{j} \cdot \vec{E}$

- For an electromagnetic wave, the electromagnetic fields \vec{E} and \vec{B} are:
 - (a) always parallel and $|\vec{E}| = c|\vec{B}|$
 - (b) always orthogonal and $c|\vec{E}| = |\vec{B}|$
 - (c) always orthogonal and $|\vec{E}| = c|\vec{B}|$
 - (d) always parallel and $c|\vec{E}| = |\vec{B}|$
- The independent polarization vectors for an electromagnetic wave are:
 - (a) 3, two orthogonal and one parallel to the direction of the wave propagation
 - (b) 2, one orthogonal and one parallel to the direction of the wave propagation
 - (c) 2 and both orthogonal to the direction of the wave propagation
 - (d) 1 and orthogonal to the direction of the wave propagation
- The observer A, which is moving with velocity v with respect to observer B, says that B's clock is running slower than his. What does observer B says about A's clock?
 - (a) that is running faster than his
 - (b) that is running slower than his
 - (c) that is running the same
 - (d) you cannot tell because Lorentz transformation cannot be reversed

- When light travels from air into water,
 - (a) its velocity, wavelength and frequency all change
 - (b) its frequency changes, but its velocity and wavelength do not change
 - (c) its wavelength changes, but its velocity and frequency do not change
 - (d) its velocity and wavelength change, but its frequency does not change
- What is the electric field inside a conductor when there are two charges outside its surface?
 - (a) there is no electric field
 - (b) the electric field is the sum of the Coulomb fields of the two charges
 - (c) the electric field is the sum of the Coulomb fields of the two charges and that of the image charge inside the conductor
 - (d) the electric field cannot be computed, we need first to know the charge on the surface of the conductor
- The scalar product $p^{\mu}p_{\mu}$ of the 4-momentum p_{μ} is equal to
 - (a) $E^2 + c^2 |\vec{p}|^2 = m^2 c^4$
 - (b) 1
 - (c) $-E^2 + c^2 |\vec{p}|^2 = -m^2 c^4$
 - (d) none of these

PROBLEM 1.

A SPHERE of radius r_0 carries a charge Q, the charge being uniformly distributed throughout the volume of the sphere.

1. What is the charge density ρ of the sphere?

2. What is the electric field vector \vec{E} both outside and inside the sphere? <u>Hint</u>: Use Gauss' law.

3. From \vec{E} find the electric potential ϕ using the fact that $\phi \to 0$ as $r \to \infty$. <u>Hint</u>: Recall that $\vec{E} = -\vec{\nabla}\phi$ implies $\phi = \int_r^\infty \vec{E} \cdot d\vec{s}$. PROBLEM 2.

A RECTANGULAR COIL of dimensions a and b and resistence R moves with constant velocity v into a magnetic field \vec{B} which is orthogonal to the plane of motion.

1. Derive an expression for the force on the coil in terms of the given parameters.

2. What is the direction of the force with respect to the direction of v?

PROBLEM 3.

T his is a problem about the transformation properties of charges, currents and electromagnetic fields.

1. Write down Maxwell's equations assuming that no dieletric or magnetic materials are presents. State your system of units.

2. If the sign of all the source charges are reversed, what appens to the electric and magnetic fields \vec{E} and \vec{B} ?

Hint: Use the Maxwell's equations you wrote and look for overall sign changes

3. If the system is space inverted, i.e. $\vec{x} \to \vec{x}' = -\vec{x}$, what happens to the charge density and current density, ρ and \vec{j} , and \vec{E} and \vec{B} ?

4. If the system is time reversed, i.e. $t \to t' = -t$, what happens to ρ and \vec{j} , and \vec{E} and \vec{B} ?